SCHEDULING AND COMPOSING WITH RISSET ETERNAL ACCELERANDO RHYTHMS

Dan Stowell

Centre for Digital Music, Queen Mary University of London, UK
{firstname.lastname}@eecs.qmul.ac.uk

ABSTRACT

Jean-Claude Risset described an "eternal accelerando" illusion, related to Shepard tones, in which a rhythm can be constructed to give the perception of continuous acceleration. The effect can in principle be derived from any rhythmic template, producing patterns with aspects of fractal self-similarity. Any attempt to use it compositionally must address the difficult issue of scheduling events (notes, beats) and structural changes in a way that integrates with the multi-layered self-similar rhythmic structure. In this paper we develop an approach to scheduling rhythms, melodies and structural changes over a Risset accelerando (or decelerando) framework. We derive the mappings which allow note sequences and sample playback to be incorporated. We discuss some compositional choices available within this framework, and demonstrate them via audio examples.

1. INTRODUCTION

The well-known "Shepard tones" audio illusion creates a set of musical notes in which the pitch relation between the notes is perceived as cyclical rather than linear, disturbing our usual sense of high vs. low pitch [4]. It achieves this by arranging the partial amplitudes within the notes so that, as we go up the scale, higher partials fade out and lower partials fade in, in such a way that after ascending one octave we have arrived at a set of partials the same as we had when playing the octave below. Jean-Claude Risset created a similar effect with a continuous upward glissando rather than individual notes [2]. He also described the rhythmic equivalent of this, in which a beat seems to accelerate continuously (although he credited Kenneth Knowlton with being the first to synthesise the effect, in 1974) [3].

The eternal accelerando is created in the same way as the eternal glissando: versions of a sound at different octaves are amplitude-weighted and combined, though using tempo octaves rather than pitch octaves. Pitch circularity has been used in a variety of compositions, both with and without the formal mapping explored by Shepard and Risset [1]. To our knowledge, Risset rhythms (as we will call them, encompassing both accelerando and decelerando) have been very little used in musical works or perceptual experiments. The exploitation of Risset rhythms may be held back by the complexities of managing accelerating fractal rhythms within compositional and sound-design environments, and combining them with non-fractal sequences. This paper aims to facilitate the use of Risset rhythms in composition by deriving mathematical expressions used to map ordinary rhythms into repeatable Risset rhythms, and exploring some of the consequences of working in a framework underpinned by Risset rhythmic structure.

Supported by EPSRC grant EP/I001832/1

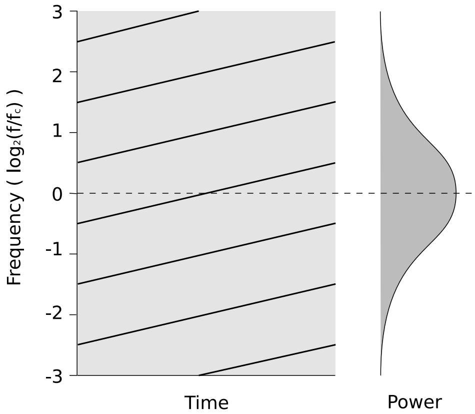

Figure 1. Diagrammatic representation of a Risset glissando ascending one octave. The diagonal lines represent individual partials, each of whose power is frequency-dependent as indicated by the bell curve on the right.

We first describe the construction of a Risset rhythm, and develop mathematically our approach which leads to convenient scheduling of such patterns, giving examples of the rhythms thus created. We then address compositional factors at both the micro and macro level, such as the use of pitch within Risset rhythms and scheduling transitions.

2. CONSTRUCTING RISSET RHYTHMS

Figure 1 outlines the elements used to synthesise a Risset glissando sound. The glissando is composed of sinusoidal partials, with their time-varying power directly mapped from their frequency. In the diagram the frequencies are shown relative to some centre frequency f_c, and the power is mapped to a cosine envelope centred on f_c and having a range of 6 octaves. The octave range is a free parameter, but should be wide enough to create a rich sound, and also that the loss of the partials fading out at the top of the range (and the gain of those fading in at the bottom of the range) is not noticeable. In our diagram the glissando performs a smooth exponential rise in frequency over time – any rise could be used, though an exponential curve is the natural choice for a continuous smooth perceived rise. After the glissando has covered an octave, the set of partial frequencies and powers is the same as that at the beginning.

The Risset accelerando is created in a similar fashion. Instead of sinusoidal partials, we have parallel streams of a rhythmic pattern played at different relative tempos, again using a smooth power mapping that ensures the streams at the centre of the tempo range are dominant. The octave relationship between the streams is again used, but the streams generally run on a slower timescale – f_c being a bpm value rather than an audible frequency. (We will in fact use r_c to denote the central rate in a tempo scale.)

We note that the eternal accelerando might be considered a generalisation of the eternal glissando: the glissando could be created via an eternal accelerando in which the sequence to be accelerated is samples in a sine wavetable.

2.1. Phase alignment and the metabar

In the glissando effect, phase alignment of the partials is generally not controlled: the fixed octave ratio between partials will generally maintain some coordination in their phases, but there is little reason to consider whether the phases of the sinusoids have completed an integer number of cycles (returned to their initial phase) by the time an octave glissando is complete. However, in the accelerando this takes on much more importance, since an offset in phase (i.e. position within the bar) is easily perceivable in many patterns. Figure 2 depicts an example accelerando, indicating the phase of each stream as it accelerates. Naturally, the phase advances faster at higher tempo: in our diagram, the streams all begin at phase zero and evolve at different rates, though the octave relationship means they come back into sync each time the slowest stream has completed a full bar.

This period may be considered a type of "bar length" for the Risset rhythm. However there is a second important period when using a consistent rate of acceleration, namely the period over which the pattern accelerates by one tempo octave. After this period, the set of stream tempos will have the same numerical values as at the start.

There is no necessary relation between the period over which the tempo doubles (the *doubling period*) and the period over which the stream phases come back into sync. However, if we constrain these periods to be equal we obtain a particular advantage which makes the Risset rhythm more amenable to work with for scheduling and composing. Under such a constraint, the pattern returns to its exact starting state (in both tempo and phase) after a fixed period, meaning the pattern can be described as a single repeating unit (see Figure 2). This unit can be repeated as can ordinary bar-length patterns – for example, audio loops can be produced and re-used.

In the next section we will develop some of the formal relationships implied by locking together the tempo-doubling period and the elapsed-phase period. To clarify the discussion, we will use the term *bar* to refer to a bar in an ordinary fixed-tempo pattern which might be used as source material to produce a Risset rhythm; when the pattern is transformed into a Risset rhythm, we will use the term *metabar* to refer to the repeating unit of accelerating rhythm, which will be a loopable sequence of events producing a perception of ever-accelerating (or decelerating) rhythm over which the perceived tempo doubles.

2.2. Mapping between bar and metabar

Let T be the duration of a bar of source material (such as an audio clip or beat pattern) to be mapped into a Risset rhythm, and τ be the duration of the metabar that will be produced. Denote by t_l the "logical time" meaning a time index into the source material, and t_e the "elapsed time" meaning a time index into the metabar. When $t_e = 0$, $t_l = 0$. The rate r is the time-variable playback rate of a pattern, $r = \frac{dt_l}{dt_e}$ – at rate 1 a pattern would play back at its normal speed. In order to distinguish between the different octave-related parallel streams in a metabar, let v be the "level" of a stream, where 0 is the level at which one bar elapses during a metabar, 1 is the level at which two bars elapse during a metabar, 2 is the level at which four bars elapse during a metabar, and so forth (cf. Figure 2).

Given the fixed exponential tempo modulation, doubling over period τ, we can express the playback rate of a stream as a function of the elapsed time:

$$r_{v,t_e} = r_{v,0} \cdot 2^{t_e/\tau} \tag{1}$$

Combining this with $r = \frac{dt_l}{dt_e}$ and the constraint that after τ elapsed time the number of bars completed (i.e. t_l/T) is 2^v, we can integrate over the range zero to τ and then apply algebra to obtain

$$r_{v,t_e} = \frac{T \log_e 2}{\tau} \cdot 2^{(t_e/\tau)+v} \tag{2}$$

The power of each stream is modulated according to its playback rate, using a cosine envelope:

$$p_r = \frac{1}{2}\left(\cos\left[\frac{\pi b}{2}\log_2\frac{r}{r_c}\right]_{-\pi}^{\pi} + 1\right) \tag{3}$$

where b is the bandwidth in octaves, and r_c is the centre rate – which might often be chosen to be 1, so that the "normal" playback rate tends to dominate the pattern produced. The square bracket notation here means that the expression within is clipped to the $\pm\pi$ range.

Equations (2) and (3) are sufficient to map, for example, an audio clip of duration T into a Risset rhythm, using sample-playback units whose rate can be modulated. One

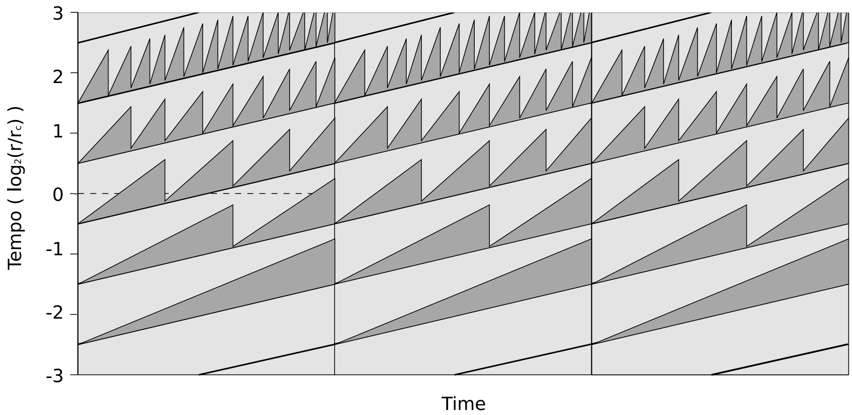

Figure 2. Diagrammatic representation of a Risset accelerando. The "partials" from Figure 1 are now parallel playback streams of the rhythm, with the vertical axis now representing tempo rather than frequency, but treated in the same way (for example having the same power mapping, not shown). The sawtooth superimposed on each stream indicates the phase within a bar. In this instance, the three boxes (metabars) are the same, and the phases continue correctly across the metabar-lines: this only happens if the metabar length, bar length, and tempo are appropriately related – see text for details.

uses a sample-playback unit for each desired level, modulating the rate and amplitude, and then sums the audio outputs.

However, these mappings are not directly useful for converting a set of events such as a melody or notated rhythm into the events required to construct the Risset-rhythm version. For this, we must integrate and invert the mapping so that bar offsets t_e can be used as input, and we must also accommodate the one-to-many nature of the mapping: when $v > 0$, multiple bars elapse during a metabar – each event in the input pattern corresponds to 2^v events to be scheduled in the output. This results in

$$t_{ev,t_l} = \tau \left(\log_2 \frac{t_l + wT + 2^v}{T} - v \right), \forall w \in 0, 1, ...(2^v - 1) \tag{4}$$

where w is a dummy variable used to iterate over the instances of the bar within one metabar.

As examples of the mapping in action, we have generated Risset rhythm from a breakbeat sample, a simple rhythm sequence and a simple tonal sequence, using (2) (3) and (4).[1]

Having established the basis for mapping from patterns or audio clips to Risset-rhythm metabars which can be looped and scheduled, we briefly consider some compositional consequences which follow when using this framework.

3. COMPOSITIONAL FACTORS

3.1. Metabar-level considerations

One consideration for the input patterns is of sparsity: relatively sparse input patterns are generally preferred, since the multi-level layering will increase their density. The bandwidth b can be manipulated to some extent to control the sparsity of the output, though too narrow a bandwidth will lessen the perceptual eternal accelerando effect, so manipulating b does not provide much flexibility. When using pitched sounds, there are further useful free choices available to the composer, as we next demonstrate.

3.1.1. Pitch warping within patterns

The original Risset rhythm used pulses at a constant pitch to create the rhythm. However, if we were to build a rhythm by manipulating a tape recording to play at variable rates, the pitches of the events would be modulated in line with the tempo. This highlights a free choice available when working with Risset rhythms: pitches of events may be modulated along with tempo, or not. Further, if modulating pitch, the events could also have gradual pitch-bends over time, or they could remain at their starting pitch. In the audio examples we show how this choice affects a rhythmic and a melodic Risset rhythm made from simple source material.[1]

The examples demonstrate that pitch modulation gives a clearer separation of the streams giving a stronger percept of rhythmic structure. Yet for melodic patterns the drawback of simple pitch modulation is obvious: notes are warped away from whichever scale is in use, generally giving an out-of-tune sound or at least falling outside a fixed key. To address this, the notes could be coerced to the nearest frequency in a scale, yielding different but related pitch patterns, as the melody straddles different scale degrees as the pattern progresses. We demonstrate coercion to the chromatic scale in our audio examples. This approach creates a musically useful variant of the technique: it enables the streams to have some degree of perceptual separation since pitches do not always overlap, yet allows a melodic/harmonic result that is not constrained to sound like varispeed tape playback.

[1] `http://archive.org/details/risset1101`

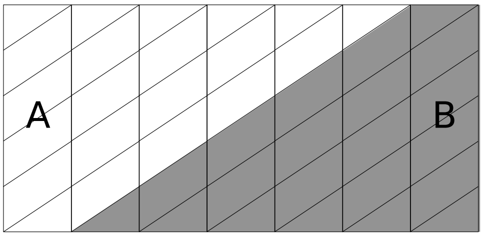

Figure 3. Perceptual crossfade from Risset rhythm A to B.

3.1.2. Placing the centroid

Above we suggested that the centre rate r_c for a Risset rhythm's power mapping could be the standard playback rate $r_c = 1$. In cases involving pitch-warping – such as sample-based playback – the spectral characteristics of the resulting rhythm can be approximated by convolving the amplitude spectrum of the input pattern with the amplitude envelope corresponding to (3). The spectral centroid of the output can thus be altered by modulating r_c, with the centroid staying close to that of the input when $r_c = 1$. Modulating r_c can be used to control a Risset rhythm's spectral placement in a mix in relation to other elements, independently of τ.

3.2. Macro-level considerations

One macro-level consideration that emerges from our approach (implied by Figure 2) is that the metabar becomes the primary compositional unit rather than the bar. This means that the composer no longer selects a tempo for the music but rather a tempo-doubling rate ($1/\tau$). The Risset construction also has consequences for other macro-level considerations. We consider crossfading, and the relation of accelerando to musical tension.

3.2.1. Crossfading Risset rhythms

Transitions between sections in a piece of music can be jump-cut or crossfade (instant or gradual). For jump-cut transitions it will often make sense to place the boundary at a metabar boundary, as in ordinary composition. For crossfades, the Risset rhythm structure provides a natural way to create a perceptual crossfade between rhythms, avoiding the audible artifact of the transitional period being made to contain more-but-quieter events than normal (as would occur in a naive crossfade). The approach is depicted in Figure 3 and crossfades by introducing the second pattern in the new streams being added in. The crossfade is not complete until all the octaves have filled out with the new content, which will take as many metabars as there are audible octaves. We give an audio example.[1]

3.2.2. Musical tension/arousal

Accelerando is typically connected with an increase in energy (arousal) in a composition, and decelerando with its decrease; the use of these tempo changes is well understood by composers. Yet the Risset rhythm is unusual compared against other tempo changes, and not just because the tempo change can continue indefinitely. Conventional accelerando includes objective changes such as an increased temporal density of events, whereas the density in a Risset rhythm stays approximately constant irrespective of accelerando or decelerando. The psychoacoustics of these tempo factors has not been teased apart, and Risset rhythm constructions could help in this; similarly the compositional consequences for arousal/tension are yet to be explored. In our experience (which listeners may confirm from the audio examples) the constant tempo change initially conveys an ordinary impression of change, but the listener can accommodate if the rhythm continues indefinitely. Compositional use of such rhythms would likely be able to make use of this to manipulate listeners' expectations.

4. CONCLUSIONS

The Risset rhythm effect has unique auditory characteristics. In this paper we have developed an approach which makes Risset rhythms amenable to creation from a variety of source material. We hope this facilitates future psychoacoustic and musical exploration of these tempo effects.

5. REFERENCES

[1] I. Braus, "Retracing one's steps: An overview of pitch circularity and Shepard tones in european music, 1550–1990," *Music Perception*, vol. 12, no. 3, pp. 323–351, 1995.

[2] J. C. Risset, "Paradoxes de hauteur: Le concept de hauteur sonore n'est pas le meme pour tout le monde," in *Proceedings of the Seventh International Congress of Acoustics Budapest*, 1971, pp. 613–616.

[3] ——, "Pitch and rhythm paradoxes: comments on "auditory paradox based on fractal waveform"," *Journal of the Acoustical Society of America*, vol. 80, no. 3, pp. 961–962, 1986.

[4] R. N. Shepard, "Circularity in judgments of relative pitch," *Psychological Review*, vol. 36, no. 12, pp. 2346–2353, Dec 1964.